

A New Approach of Finding Local Extrema in Grayscale Images based on Space-Oriented Masks

Oday Jasim Mohammed Al-Furaiji, Anh Tuan Nguyen, and Viktor Yurevich Tsviatkou

Abstract—The aim of the work is to develop an algorithm for extracting local extrema of images with low computational complexity and high accuracy. The known algorithms for block search for local extrema have low computational complexity, but only strict maxima and minima are distinguished without errors. The morphological search gives accurate results, in which the extreme areas are formed by non-strict extrema, however, it has high computational complexity. This paper proposes a block-segment search algorithm for local extrema of images based on space-oriented masks. The essence of the algorithm is to search for single-pixel local extrema and regions of uniform brightness, comparing the values of their boundary pixels with the values of the corresponding pixels of adjacent regions: the region is a local maximum (minimum) if the values of all its boundary pixels are larger (smaller) or equal to the values of all adjacent pixels. The developed algorithm, as well as the morphological search algorithm, allow to detect all single-pixel local extrema, as well as extreme areas, which exceeds the block search algorithms. At the same time, the developed algorithm in comparison with the morphological search algorithm requires much less time and RAM.

Index Terms—local extrema of images, strict and non-strict extrema, block-segment search for local extrema, image segmentation, space-oriented masks, region growing

I. INTRODUCTION

Non-Maximum Suppression (NMS) is the task of finding all local maxima in an image. The term ‘non-maximum suppression’ is first appeared in an edge detection context as a method to reduce thick edge responses to thin lines [1]. This type of directional NMS operates one dimensionally (1-D) perpendicular to the edge. Kitchen and Rosenfeld [2] extended the concept to isotropic NMS to locate two-dimensional (2-D) feature points from an image. The feature points are selected as local maxima of a ‘cornerness’ image over some neighborhood. This NMS approach to corner detection was subsequently adopted by many interest point detectors [3]–[5].

Image processing often requires the determination of initial elements, which can be local 2-D extrema (local maxima and local minima). To search for local extrema, some block algorithms [6]–[12] and morphological algorithms [13] are used.

In block algorithms, the search for extrema is computed within overlapping blocks, usually 3×3 or $(2n+1) \times (2n+1)$ pixels in size. Such algorithms have low computational complexity, but they are characterized by over-processing (when selecting local minimum pixels that are local maxima are reprocessed, and vice versa); skipping non-strict extrema (for a homogeneous region consisting of several adjacent pixels with the same values, none of the pixels in this region is detected as a local extremum (Fig. 1)); errors in the search for local extreme regions (if, for example, the image contains a homogeneous region, some pixels of which have the brightness values are greater than or equal to the brightness values of adjacent pixels, but other pixels of the region have the brightness values are less than the brightness values of adjacent pixels. The first case is erroneously detected as local maxima but the second case is exactly detected as non-maxima).

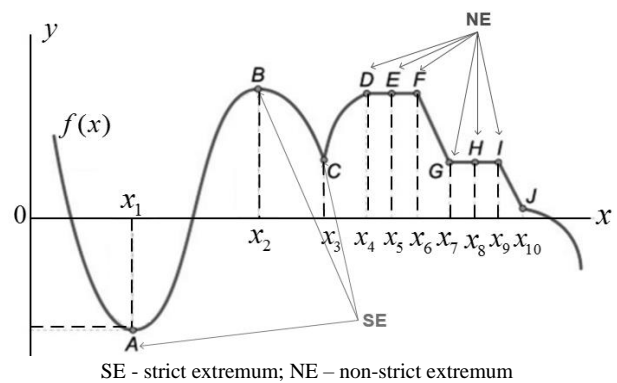


Fig. 1. Types of local extrema in one-dimensional representation

Finding local extrema (maxima and minima) is often solved by mathematical morphology using dilation and erosion operations, respectively [13]. It gives accurate results compared to block algorithms, highlighting both strict extrema and extreme areas (multi-pixel extrema) formed by non-strict extrema. However, the morphological algorithm has high computational complexity, which is associated with separate processing of maxima and minima, as well as iterative processing of the neighborhoods of all pixels.

In this paper, the aim of the work is to develop an algorithm for extracting local extrema in grayscale images

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with low computational complexity, high accuracy and less RAM.

II. RELATED WORKS

From the related NMS algorithms [6]–[12], an extended algorithm to find local extrema (local maximum and local minimum) in an image $f(y, x)$ can be described by the following:

$$e_{MAX}(y, x) = \begin{cases} 1, & \forall y' \forall x' (f(y, x) > f(y + y', x + x')) \\ 0, & \forall y' \forall x' (f(y, x) \leq f(y + y', x + x')) \end{cases} \quad (1)$$

$$e_{MIN}(y, x) = \begin{cases} 1, & \forall y' \forall x' (f(y, x) < f(y + y', x + x')) \\ 0, & \forall y' \forall x' (f(y, x) \geq f(y + y', x + x')) \end{cases} \quad (2)$$

where $y = \overline{0, Y-1}$, $x = \overline{0, X-1}$, $(y' \in \{-1, 0, 1\}) \wedge (x' \in \{-1, 0, 1\}) \setminus ((y' = 0) \wedge (x' = 0))$. As a result of combining the matrices E_{MAX} and E_{MIN} , a matrix of local extrema E is obtained, in which the values of the elements are calculated using the expression:

$$e(y, x) = e_{MAX}(y, x) + e_{MIN}(y, x) \quad (3)$$

Regional extrema are detected better by morphological reconstruction by dilation and erosion [13], in which a *regional minimum* M of an image f at elevation t is a connected component of pixels with the value t whose external boundary pixels have a value strictly greater than t . M is a *regional minimum* at level $t \Leftrightarrow M$ is connected and

$$\begin{cases} \forall p \in M, f(p) = t, \\ \forall q \in \delta^{(1)}(M) \setminus M, f(q) > t. \end{cases} \quad (4)$$

Similarly, a *regional maximum* M of an image f at elevation t is a connected component of pixels with the value t whose external boundary pixels have a value strictly less than t . M is a *regional maximum* at level $t \Leftrightarrow M$ is connected and

$$\begin{cases} \forall p \in M, f(p) = t, \\ \forall q \in \delta^{(1)}(M) \setminus M, f(q) < t. \end{cases} \quad (5)$$

The regional extrema of an image are defined as the union of its regional minima and maxima. According to (5), the set of all maxima of an image f at level t corresponds to the connected components of the cross-section of f at level t that are not connected to any component of the cross-section of f at level $t+1$. They are therefore not reconstructed by the morphological reconstruction by dilation $CS_t(f)$ from $CS_{t+1}(f)$. The regional maxima of an image f at level t can be written by the following:

$$\begin{aligned} RMAX_t(f) &= RMAX(f) \cap T_t(f) = \\ &= CS_t(f) \setminus R_{CS_t(f)}^\delta[CS_{t+1}(f)]. \end{aligned} \quad (6)$$

The set of all maxima is defined by considering the union of the maxima obtained at each level t :

$$RMAX_t(f) = \bigcup_t \{CS_t(f) \setminus R_{CS_t(f)}^\delta[CS_{t+1}(f)]\}. \quad (7)$$

Since $CS_{t+1}(f) = CS_t(f-1)$ and

$RMAX_t(f) \cap RMAX_{t'}(f) = \emptyset$ for all $t \neq t'$, the set difference in (6) can be replaced with an algebraic difference and the union by a summation. The summation is then distributed and the threshold superposition principle gives:

$$RMAX(f) = f - R_f^\delta(f-1) \quad (8)$$

If the image data type does not support negative values, the following equivalent definition must be considered:

$$RMAX(f) = f + 1 - R_{f+1}^\delta(f) \quad (9)$$

Similarly, the regional minima of an image f at level t are denoted by

$$\begin{aligned} RMIN_t(f) : RMIN_t(f) &= RMIN(f) \cap T_t(f) = \\ &= R_{CS_{t+1}(f)}^\epsilon[CS_t(f) \setminus CS_{t+1}(f)]. \end{aligned} \quad (10)$$

The set of all regional minima is denoted by $RMIN$ and defined by threshold superposition:

$$RMIN(f) = R_f^\epsilon(f+1) - f \quad (11)$$

From (1)–(3) the main disadvantages of block algorithms for finding local extrema are the following:

1) Redundancy of processing. The independent formation of matrices by (1) also leads to redundancy of processing, since the matrix pixels are re-processed when the matrix is formed to find the local minimum.

2) Skipping non-strict extrema. From (1) it follows that if the matrix has a local maximum homogeneous region consisting of several adjacent pixels with the same values (non-strict maxima), none of the pixels in this region is detected as a local maximum. The same is true for local minima by (2). The number and area of such regions grow during quantization, filtering, and restoration of images after compression. In such cases, skipping non-strict extrema leads to incomplete image segmentation, errors in detection, localization, and parameterization of objects.

3) The need for additional processing of the resulting matrix to assign label numbers to local extrema.

From (4)–(11), the main disadvantages of morphological algorithms for finding local extrema are the following:

1) Separate processing of local maxima and local minima.
2) Iterative processing of neighborhoods of all image pixels.

3) The need for additional segmentation for assigning label numbers to extreme regions.

III. PROPOSED ALGORITHM

To eliminate the above disadvantages a new fast non-maximum suppression based on space-oriented masks (SOM – Space-Oriented Masks) and its mathematical model (12) to extract all local extrema in grayscale images with low computational complexity and high accuracy are proposed. The essence of the algorithm is to search for single-pixel local extrema and areas of uniform brightness, comparing the values of their boundary pixels with the values of the corresponding pixels of adjacent areas by following: the region is a local maximum (minimum) if the values of all its boundary pixels are greater (less) or equal to the values of all adjacent pixels. Along with single-pixel extrema, the algorithm takes into account homogeneous regions (of two or more pixels), which are local maxima or minima with respect to adjacent regions due to image

segmentation using SOM in Fig. 2 and analysis of brightness changes at the boundaries of the regions.

$$e(y,x) = \begin{cases} -n_E, & (\forall p(y,x) \in M, f(p) = t) \wedge \\ & \wedge (\forall q(y,x) \in \delta^{(1)}(M) \setminus M, f(q) > t), \\ n_E, & (\forall p(y,x) \in M, f(p) = t) \wedge \\ & \wedge (\forall q(y,x) \in \delta^{(1)}(M) \setminus M, f(q) < t), \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

The values of the elements $e(y,x)$ of the matrix E of local extrema indicate that the corresponding pixels of the image belong to the n_E -th maximum ($e(y,x) = n_E$), minimum ($e(y,x) = -n_E$), or non-extremum ($e(y,x) = 0$) of region $R(n_E)$, $n_E \in [1, N_E]$, N_E – the total number of local extrema. The proposed algorithm allows high accuracy in comparison with block algorithms [6]–[12] and less computational complexity in comparison with morphological algorithm [13] by the following:

- 1) Combined search for maxima and minima by assigning elements to the matrix $e(y,x)$ both positive and negative values;
- 2) Taking into account non-strict extrema by estimating the neighborhood of an image pixel $f(y,x)$ using non-strict inequalities;
- 3) The elimination of search errors of local extreme regions due to the assessment of the neighborhoods of all pixels of each homogeneous region $R(n_E)$;
- 4) The absence of the need for segmentation of the matrix E of local extrema due to the assignment of numbers to single-pixel extrema and extreme regions;
- 5) One-time processing of neighborhoods of all pixels by growing homogeneous regions $R(n_E)$.

The algorithm begins with an initialization block. Then, in the loop, the next non-segmented and unblocked pixel is searched. If an unblocked probable maximum is found, then it is block-checked for a strict and non-strict maximum. If the current pixel is not a strict or non-strict maximum, then the corresponding adjacent probable minimum is blocked (if the current pixel is less than the adjacent one, then the adjacent pixel cannot be a minimum) and the processing proceeds to the unblocked probable minimum. If the current pixel is a strict or non-strict maximum, then all adjacent probable maxima are blocked and the current pixel is checked for a strict maximum.

If the condition is met, then the current pixel is registered as a single-pixel local maximum and the processing of the next pixel is completed. If the condition is not met, then the segmentation of the probable local maximum region using SOM (Fig. 2) is carried out. To do this, a region in brightness is grown as a result of the gradual addition of neighboring pixels with equal values to the current pixel. All pixels adjacent to the boundary pixels of the formed region, which are non-strict maxima, are blocked as probable maxima. If at least one boundary pixel of the region is not a non-strict maximum, then the entire region is not a local maximum. In this case, the transition to processing the next pixel is performed. If all the boundary pixels of the region are non-strict maxima, then the selected homogeneous region is registered as a local maximum and the process

proceeds to the processing of the next pixel. Unblocked probable minima are processed similarly. If all the pixels are not processed, then the transition to the search for the next non-segmented and unblocked pixel is performed. If all the pixels are processed, then the resulting matrix of local extrema is formed.

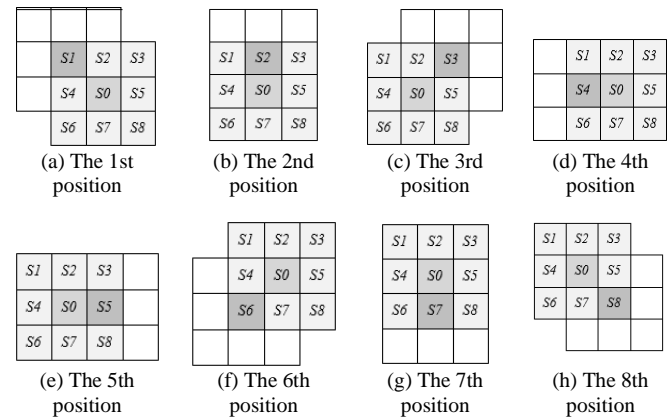


Fig. 2. Space-oriented masks and region growing positions

As a result of this algorithm, a matrix of extrema is formed, in which the value of each element indicates the label of the extrema or its absence. This data is used for further image processing.

IV. RESULTS AND ANALYSIS

The proposed algorithm SOM (E) is compared with some other known algorithms: straightforward algorithm (A) [9], [11], spiral scan order (B) [10], scanline3x3 algorithm (C) [11] and gray-scale morphology algorithm (D) [13]. Besides, the Matlab's built-in functions *imregionalmax*, *imregionalmin*, *imdilate* and *imerode* were used for the morphology implementation using Matlab ®Image Processing Toolbox (R2015b).

Six gray-scale images were used in this experiment: worst and best are 256×256 images of worst and best-case scenarios for the straightforward algorithm; random is a 256×256 image of uniformly distributed noise; 256×256 , 512×512 and 1024×1024 refer respectively to Harris corner [3] images of the Brain, Lena and City square image of the corresponding size (Fig. 3).

The execution time shown in Fig. 4b is generally in agreement with the algorithm complexity in Fig. 4a. On average, our method runs at a speed of $0.63 \mu\text{s}$ per pixel. This is 3.15 times faster than the morphology method.

Moreover, our algorithm implemented in pure Matlab code is almost faster than Matlab's built-in functions *imdilate* and *imerode* written in C++. The execution time of our algorithm is quite stable over a wide range of image contents and sizes. It is also independent of the number of detected local extrema. The proposed algorithm can run faster for finding local extrema in low-frequency images (Fig. 3d).

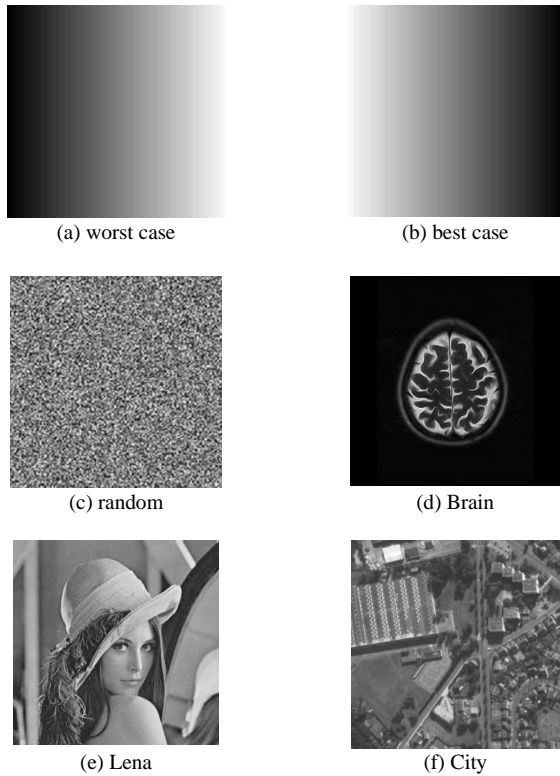
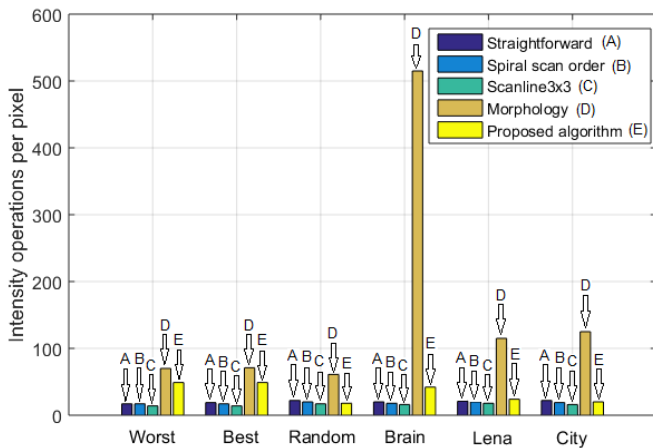
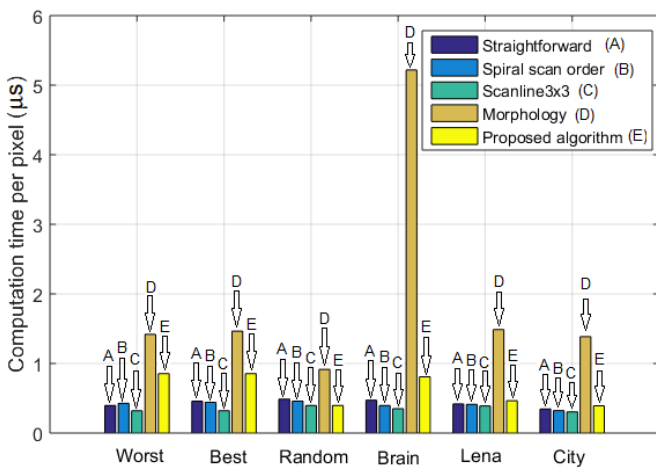


Fig. 3. Test grayscale images



(a) Number of operations per pixel



(b) Computation time per pixel

Fig. 4. Comparison of different algorithms for 3×3 non maximum suppression using Matlab2015b

From Fig. 4 the proposed algorithm SOM (E) is compared with well-known algorithms of the block search Scanline3x3 (C) [11] and morphological search (D) [13] for local extrema by the number of extrema, processing speed, and memory costs. The results were obtained with averaging over images in size 512×512 pixels divided into 15 types depending on the shape of the histograms of their brightness (Fig. 5, Fig. 7, Table I).

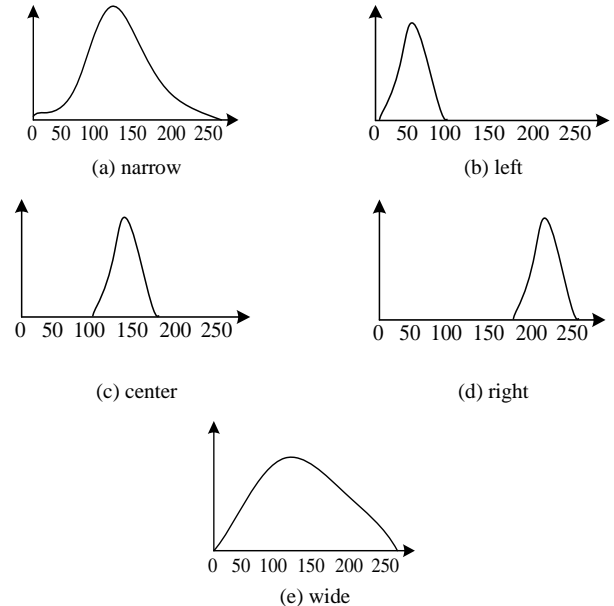


Fig. 5. Histograms of test images

It was experimentally established that the proposed and morphological algorithms extract in 1.4 times more local extrema compared to the Scanline3×3 algorithm by taking into account non-strict extrema. The selection of extreme regions in addition to strict extrema leads to an increase in the computational complexity of the proposed algorithm in comparison with the Scanline3×3 algorithm. In this paper, we used computing platforms Intel Core i3 3.1 GHz/6 GB of RAM/Windows 7/Matlab2015b (IWM platform) (Fig.8, 9), Intel Core i3 3.1 GHz/6 GB of RAM/Windows 7, implementation in C ++ (IWC platform), Raspberry Pi/ARM-A53/Linux, implementation in C ++ (RLC platform) (Fig. 6) to compare algorithm effectiveness. The proposed algorithm requires 2.2 (using IWM platform), 1.7 (using IWC platform) and 2.0 (using RLC platform) times more time and 5.8 times more RAM compared to the Scanline3x3 algorithm. Moreover, the proposed algorithm requires 4.1 (using IWM platform) and 3.1 (using IWC platform) times less time and 2.1 times less RAM compared to morphological algorithm.



Fig. 6. Implementation on Raspberry Pi 3 Model B

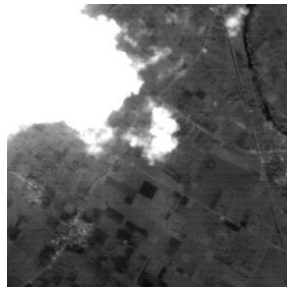


Image 1a

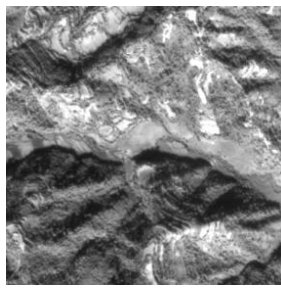


Image 2a

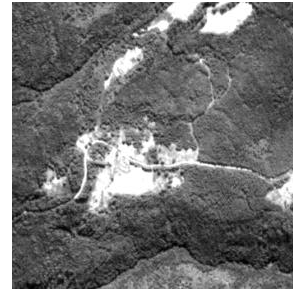


Image 3a

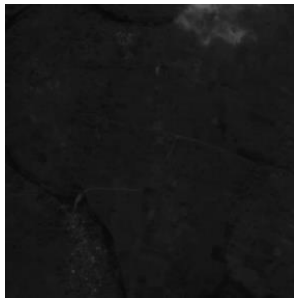


Image 1b



Image 2b

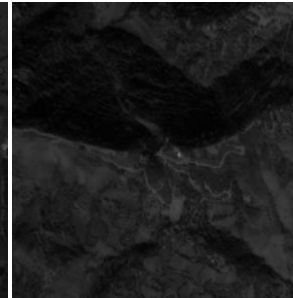


Image 3b

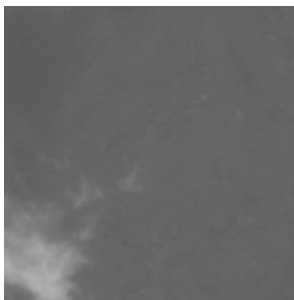


Image 1c

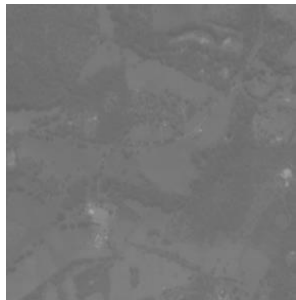


Image 2c

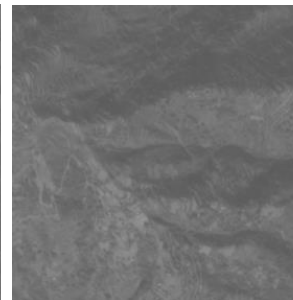


Image 3c

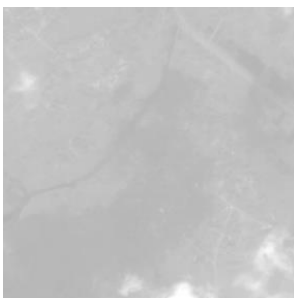


Image 1d

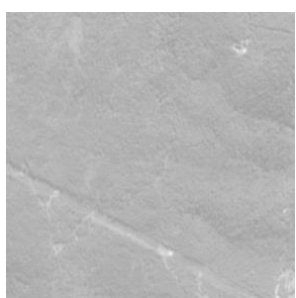


Image 2d

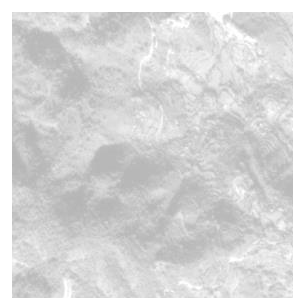


Image 3d



Image 1e

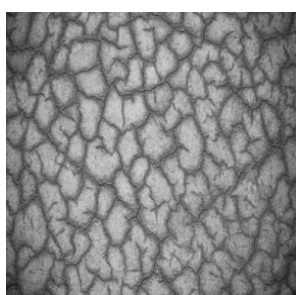


Image 2e

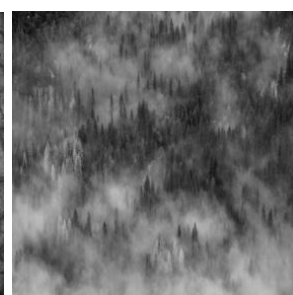


Image 3e

Fig. 7. Test grayscale images from different histograms. Images 1 to 15 from left to right and top to bottom

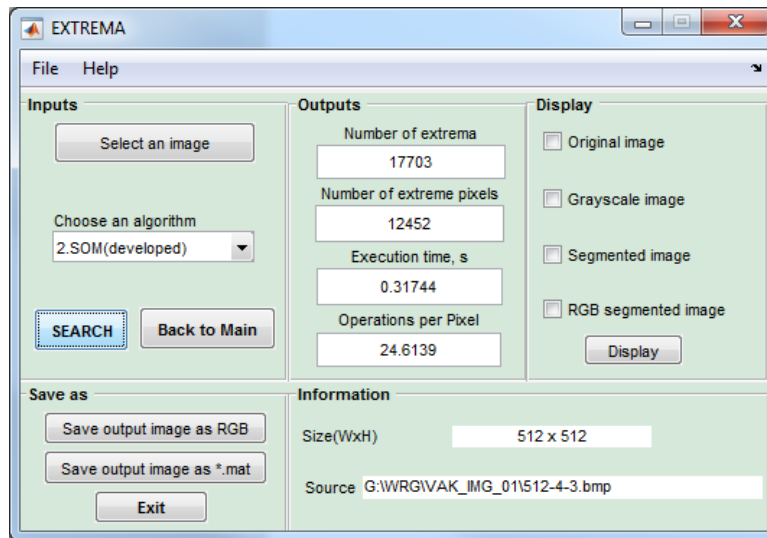


Fig. 8. User interfaces of proposed algorithms using MATLAB2015b



Image 1a

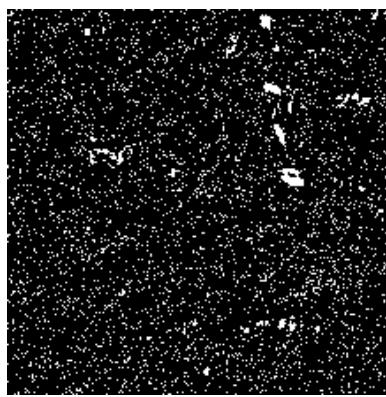


Image 2a

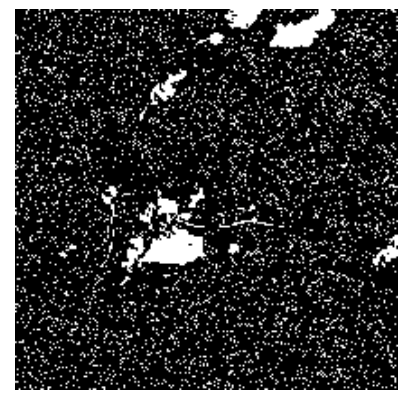


Image 3a

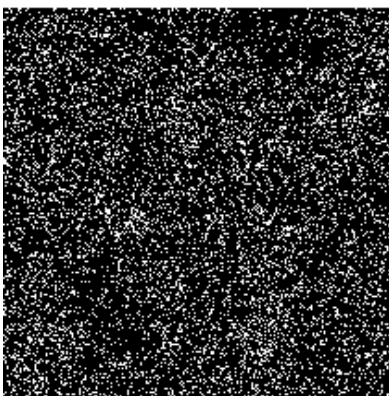


Image 1b

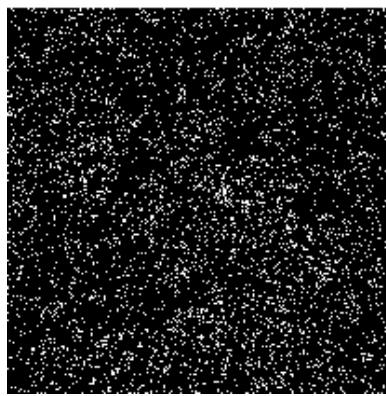


Image 2b

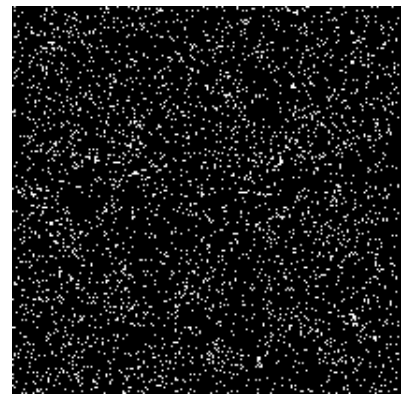


Image 3b

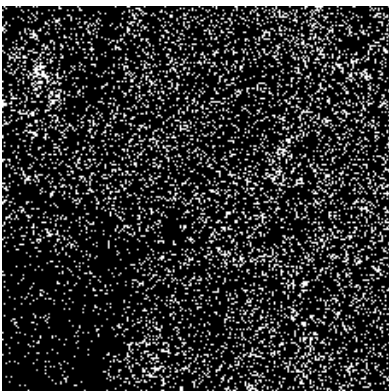


Image 1c

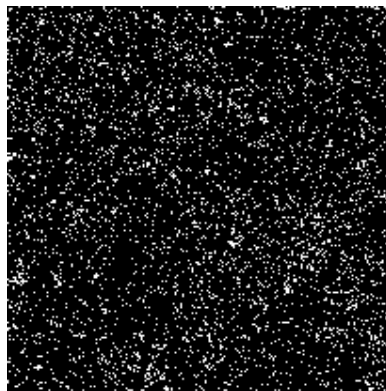


Image 2c

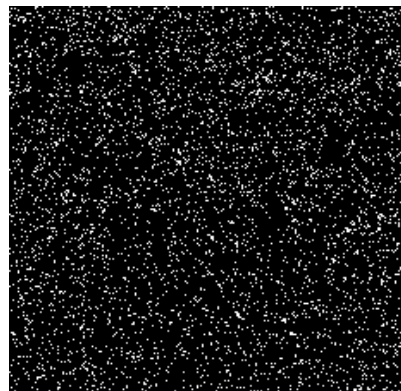


Image 3c

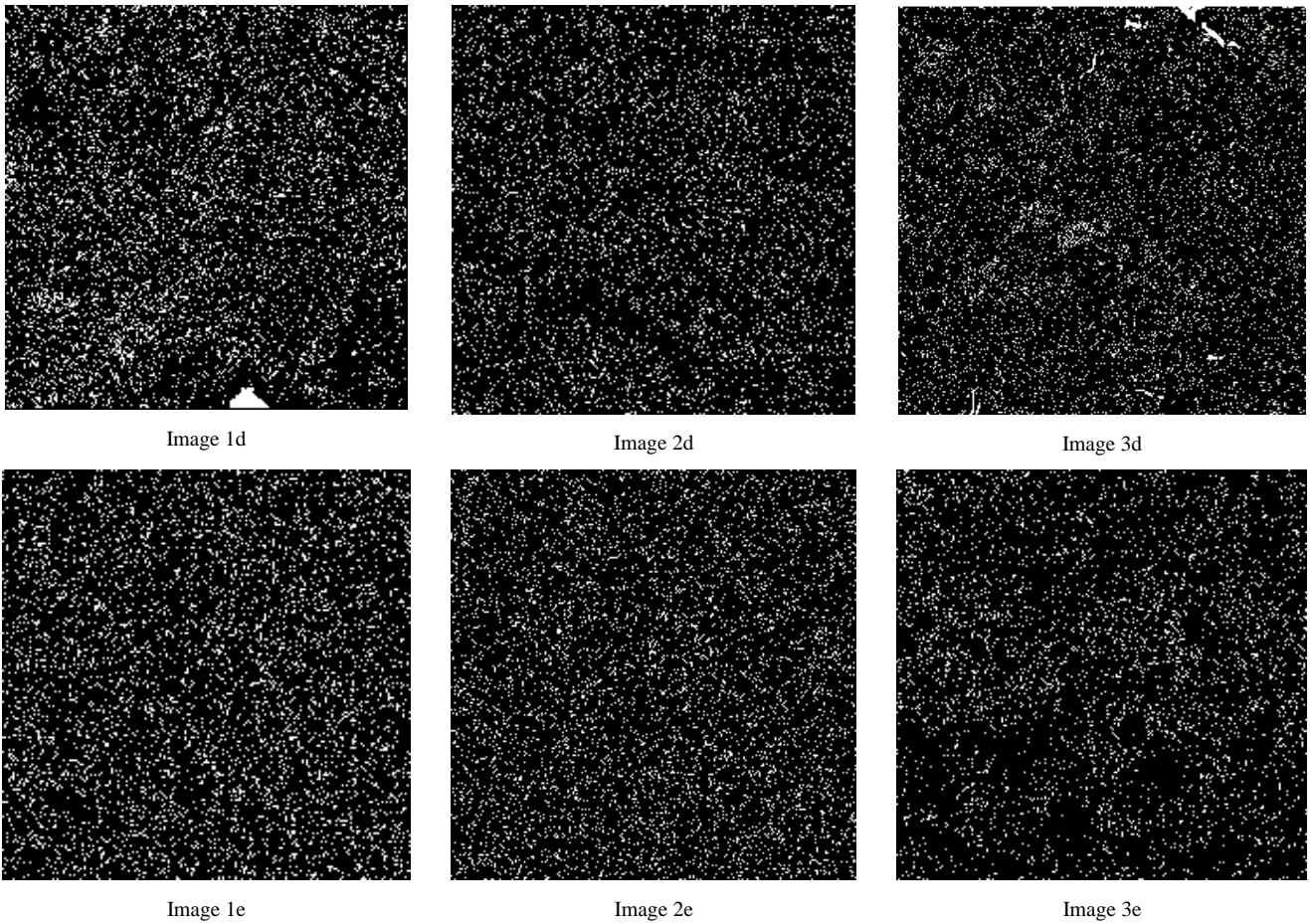


Fig. 9. Images of local extrema from above grayscale images

TABLE I
RESULTS OF FINDING LOCAL EXTREMA IN GRAYSCALE IMAGES

Image	Number of extreme pixels		Number of local extrema		Execution time, seconds								Number of Operations per Pixel		Number of missed extrema	
					IWM platform			IWC platform			RLC platform					
	C	E, D	C	E, D	C	E	D	C	E	D	C	E	C	E	C	E, D
1	16046	83053	16046	23304	0.087	0.161	1.987	0.028	0.052	0.147	0.089	0.241	16	47	7258	0
2	13091	24103	13091	16558	0.086	0.105	0.324	0.026	0.034	0.128	0.087	0.122	16	21	3467	0
3	14006	35713	14006	18575	0.085	0.109	0.403	0.027	0.037	0.129	0.087	0.135	16	23	4569	0
4	18344	44548	18344	27351	0.088	0.176	0.524	0.028	0.066	0.135	0.091	0.265	17	50	9007	0
5	9925	25433	9925	15127	0.087	0.147	0.534	0.026	0.053	0.128	0.086	0.198	16	38	5202	0
6	9260	20732	9260	13410	0.084	0.123	0.475	0.027	0.039	0.125	0.085	0.144	15	26	4150	0
7	16025	41380	16025	24137	0.088	0.183	0.782	0.028	0.067	0.132	0.089	0.263	17	50	8112	0
8	5612	24759	5612	10441	0.082	0.155	0.660	0.024	0.056	0.125	0.083	0.213	15	40	8429	0
9	8761	22494	8761	13486	0.082	0.124	0.390	0.026	0.042	0.127	0.085	0.152	15	28	4725	0
10	14461	36194	14461	20842	0.089	0.192	0.622	0.028	0.070	0.135	0.088	0.268	17	52	6381	0
11	11668	26065	11668	17059	0.084	0.122	0.432	0.026	0.038	0.130	0.086	0.137	16	25	5391	0
12	13102	25554	13102	17703	0.086	0.121	0.368	0.027	0.037	0.129	0.087	0.133	16	24	4601	0
13	28251	32573	28251	30547	0.090	0.099	0.242	0.029	0.031	0.134	0.093	0.108	17	18	2296	0
14	23473	27308	23473	25510	0.091	0.098	0.254	0.028	0.030	0.126	0.091	0.105	17	18	2037	0
15	13292	20049	13292	16269	0.087	0.104	0.275	0.027	0.033	0.122	0.087	0.116	16	20	2977	0

V. EXTENSIONS AND APPLICATIONS

The proposed algorithm can be used to find distinctive feature points in an image. To improve the repeatability of a detected corner across multiple images, an image corner is

often selected as a local extremum whose cornerness is significantly higher (or lower) than the close second highest (or lowest) peaks [4]–[5], [14].

For some applications such as multi-view image matching, an evenly distributed set of interest points for

matching is desirable. An oversupplied set of NMS feature points can be given to an adaptive non-extreme suppression process [15], which reduces cluttered corners to improve their spatial distribution.

Moreover, the proposed algorithm can be used to video de-noising: use NMS to detect highlight points in a video frame, align these points to estimate global shift and average aligned video frames to improve Signal-to-Noise Ratio. In general, the proposed algorithm enables some applications: image segmentation, image recognition, image compression, etc.

VI. CONCLUSION

A mathematical model and an algorithm for finding local extrema in grayscale images based on space-oriented masks and analysis of the brightness of adjacent pixels and regions are proposed. The proposed algorithm, as well as the morphological algorithm, allow to detect all single-pixel local extrema and extreme areas consisting of pixels with the same values. Moreover, the proposed algorithm requires about 3 to 4 times less time depending on the computing platform with averaging over image types and about 2 times less RAM compared to the morphological algorithm.

REFERENCES

- [1] A. Rosenfeld and A. Kak, *Digital Picture Processing*. Academic Press, 1976. p. 457.
- [2] L. Kitchen and A. Rosenfeld, "Gray-level corner detection," *Pattern Recognition Letters* 1 (2) (1982). pp. 92–102.
- [3] C. A. Harris and M. Stephens, "Combined corner and edge detector," *Proceeding of the Fourth Alvey Vision Conference*, Manchester, 31 August-2 September 1988, pp.147–151.
- [4] D. Lowe, "Distinctive image features from scale-invariant keypoints", *International Journal of Computer Vision (IJCV60)*, 2004, pp. 91–110.
- [5] K. Mikolajczyk and C. Schmid, "Scale and affine invariant interest point detectors", *International Journal of Computer Vision (IJCV60)*, 2004, pp. 63–86.
- [6] M. A. Van Herk, "Fast algorithm for local minimum and maximum filters on rectangular and octagonal kernels", *Pattern Recognition Letters* 13 (1992), pp. 517–521.
- [7] J. Gil and M. Werman, "Computing 2-D min, median, and max", *IEEE Trans. on PAMI* 15 (1993), pp. 504–507.
- [8] M. Brown, R. Szeliski, and S. Winder, "Multi-image matching using multi-scale oriented patches", *Proceeding of Computer Vision and Pattern Recognition (CVPR'05) 1* (2005), pp. 510–517.
- [9] A. Neubeck and L. Van Gool, "Efficient non-maximum suppression", *Proceeding of ICPR3*, 2006, pp. 850–855.
- [10] W. A. Forstner and E. Gulch, "Fast operator for detection and precise locations of distinct points, corners, and centres of circular features", *Proceeding of Intercommission Conference on Fast Processing of Photogrammetric Data*, 1987, pp. 281–305.
- [11] Tuan Q. Pham, "Non-maximum Suppression Using fewer than 2 Comparisons per Pixel", *Advanced Concepts for Intelligent Vision Systems (ACIVS) 12*, Sydney, Australia, December 13-16 (2010), pp. 438–451.
- [12] Y. Ryu, Y. Park, J. Kim, and S. Lee, "Image edge detection using fuzzy c-means and three directions image shift method", *IAENG International Journal of Computer Science* 45.1 (2018), pp. 1–6.
- [13] P. Soille, *Morphological Image Analysis: Principles and Applications*. Springer, 2006.
- [14] C. Schmid, R. Mohr, and C. Bauckhage, "Evaluation of interest point detectors," *International Journal of Computer Vision (IJCV37)*, 2000, pp. 151–172.
- [15] M. Brown, R. Szeliski and S. Winder, "Multi-image matching using multi-scale oriented patches," *Proceeding of Computer Vision and Pattern Recognition (CVPR'05) 1* (2005), pp. 510–517.



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