2-D non-separable integer implementation of paraunitary filter bank based on the quaternionic multiplier block-lifting structure

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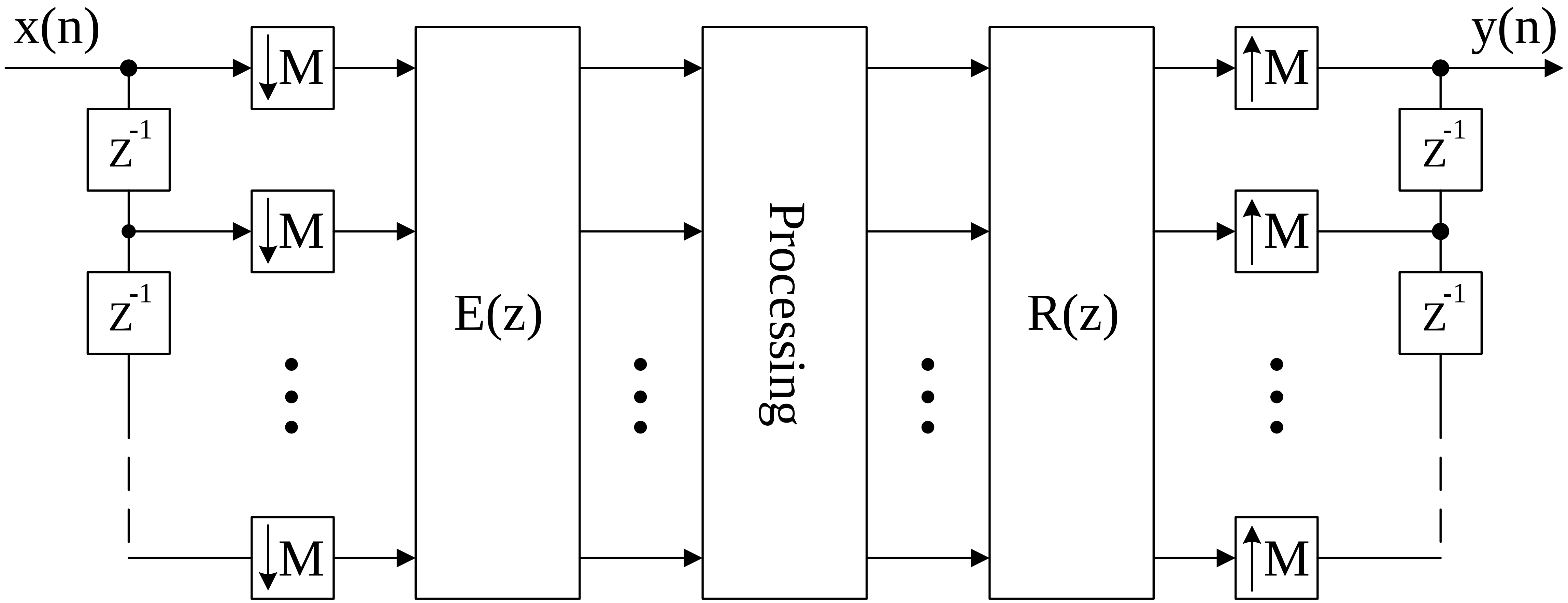
1. Introduction.

A polyphase representation of maximally decimated M-channel filter bank:



where $\mathbf{E}(z)$ and $\mathbf{R}(z)$ are analysis and synthesis transfer matrices respectively.

Biorthogonal filter bank (BOFB): $\mathbf{R}(z)\cdot\mathbf{E}(z) = bz^{-l}\mathbf{I},\ b \neq 0,\ l \geq 0.$

Paraunitary filter bank (PUFB): $\mathbf{E}^T(z^{-1})\cdot\mathbf{E}(z) = \mathbf{I}$ and $\mathbf{R}(z) = \mathbf{E}^T(z^{-1})$.

2. The main objective of this work

One-dimensional linear phase PUFB's can be applied to the construction of multidimensional separable systems. 2-D signals (images) are **separately** transformed along vertical and horizontal directions.

However, multidimensional signals are generally **non-separable**, and this approach does not exploit their characteristics **effectively**.

2-D non-separable filter banks (FBs) more effective for image coding than separable FBs, because they take into account the 2-D nature of the input signal and have better frequency characteristics.

The **research goal** is factorization of 2-D non-separable quaternionic paraunitary filer bank (2-D-NSQ-PUFB) and finite-precision FPGA implementation for L2L image coding.

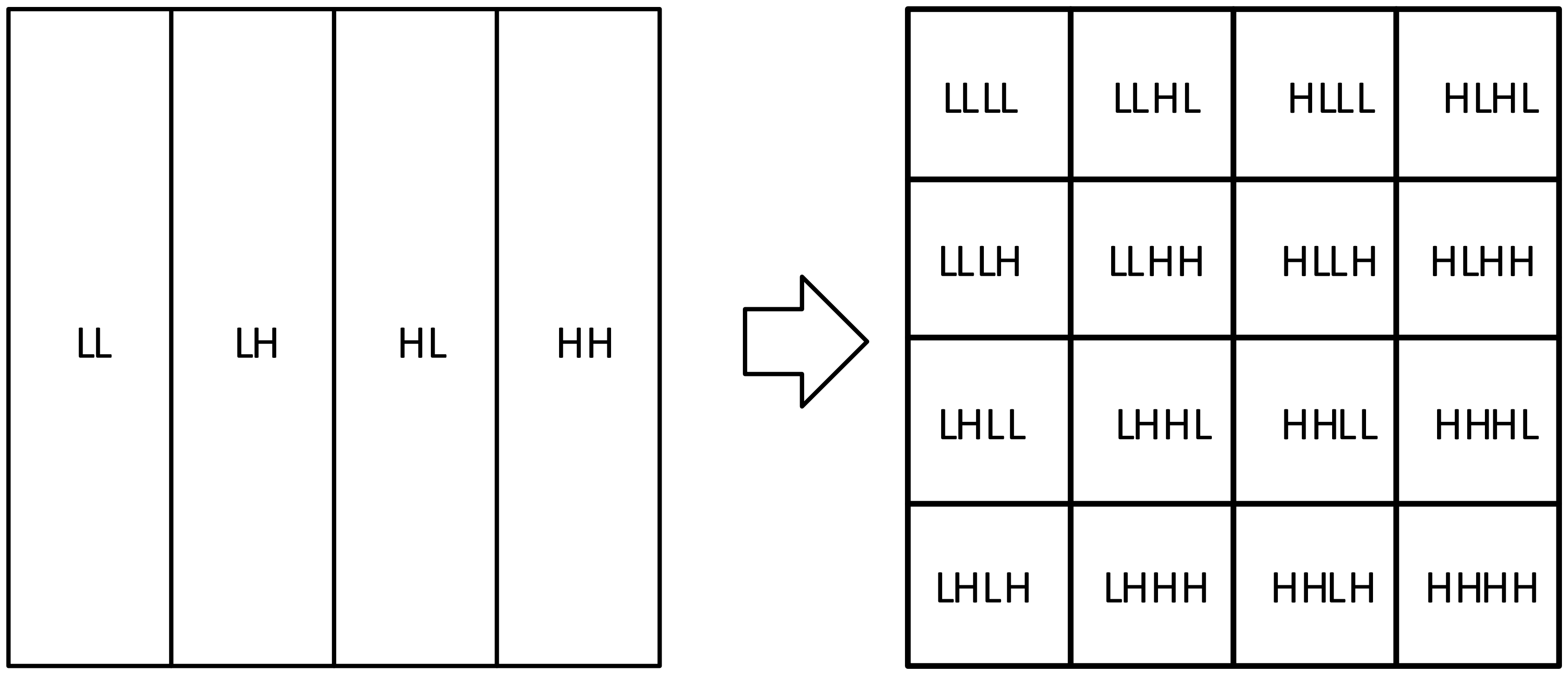
3. Separable 2-D transform

The 2-D transform based on the orthogonal transform $\Theta_{(n,n)}$ applied to 2D input signal $\mathbf{x}_{(n,n)}$ separately by column and row[1]:

$$\mathbf{y}_{n,n} = \mathbf{\Theta}_{n,n} \cdot \mathbf{x}_{n,n} \cdot \mathbf{\Theta}^{\mathrm{T}}_{n,n}$$

Intermediate result $\mathbf{x}_{n,n} \cdot \mathbf{\Theta}^{\mathrm{T}}_{n,n}$ require additional memory of size n, n.

Example for 4-channel FB



[1] N. A. Petrovsky, E. V. Rybenkov and A. A. Petrovsky, "Two-dimensional non-separable quaternionic paraunitary filter banks," *2018 Signal Processing: Algorithms, Architectures, Arrangements, and Applications (SPA)*, Poznan, 2018, pp. 120-125. **doi: 10.23919/SPA.2018.8563311**.

4. Memory-efficient high-throughput 2-D filter banks

Definition 1 (forward transform): $\mathbf{x}_{n^2,1} = \mathrm{tv}\left(\mathbf{x}_{n,n}\right)$:

$$\left[\mathrm{x}_{1,1} \cdots \mathrm{x}_{1,n} \cdots \mathrm{x}_{n,1} \cdots \mathrm{x}_{n,n}\right]^{\mathrm{T}} \xleftarrow{\mathrm{tv}} \left[\mathrm{x}_{n,n}\right]$$

Definition 2 (forward transform of the transposed matrix): $\mathbf{z}_{n^2,1} = \mathrm{tv}\left(\mathbf{x}_{n,n}^{\mathrm{T}}\right)$:

$$\left[\mathrm{x}_{1,1} \cdots \mathrm{x}_{n,1} \cdots \mathrm{x}_{1,n} \cdots \mathrm{x}_{n,n}\right]^{\mathrm{T}} \xleftarrow{\mathrm{tv}} \left[\mathrm{x}_{n,n}\right]^{\mathrm{T}}$$

The vectors $\mathbf{z}_{n^2,1}$, $\mathbf{x}_{n^2,1}$ and matrix $\mathbf{x}_{n,n}$ are related as follows:

$$\mathbf{z}_{n^2,1} = \mathrm{tv}\left(\mathbf{x}_{n,n}^{\mathrm{T}}\right) = \mathbf{P}\cdot\mathrm{tv}\left(\mathbf{x}_{n,n}\right) = \mathbf{P}\cdot\mathbf{x}_{n^2,1}$$

where $\mathbf{P}$ is permutation matrix of size $(n^2 \times n^2)$, which perform transpose in vector-matrix form.

5. Memory-efficient high-throughput 2-D filter banks

The factorization of memory efficient transform:

$$\mathbf{y}_{n^2,1} = \mathfrak{D}\left(\mathbf{\Theta}\right)\cdot\mathbf{P}\cdot\mathfrak{D}\left(\mathbf{\Theta}\right)\cdot\mathbf{P}\cdot\mathbf{x}_{n^2,1} = \ddot{\mathbf{\Theta}}_{n^2,n^2}\cdot\mathbf{x}_{n^2,1}$$

$$\mathfrak{D}\left(\mathbf{\Theta}\right) = \mathrm{diag}\left(\mathbf{\Theta},\ldots,\mathbf{\Theta}\right) = \boldsymbol{I}_n \otimes \mathbf{\Theta}_{n,n}$$

where $\otimes$ is Kronecker product; $\ddot{\mathbf{\Theta}}_{n^2,n^2}$ is 2-D transformation matrix.

The factorization of 2-D separable M-channel FB (M = 4) with polyphase matrix $\mathbf{E}(z)$:

$$\mathbf{y}_{M^2,1} = \mathfrak{D}\left(\mathbf{E}(z)\right)\cdot\mathbf{P}\cdot\mathfrak{D}\left(\mathbf{E}(z)\right)\cdot\mathbf{P}\cdot\mathbf{x}_{M^2,1}$$

2-D separable FB work with a signal of size $(M \times M)$.

6. Hypercomplex algebra

The Quaternion algebra

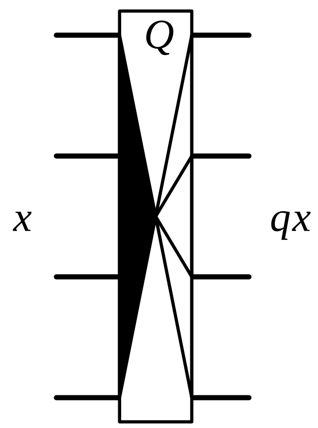
The quaternion algebra $\mathbb{H}$ is an associative non-commutative four-dimensional algebra $\mathbb{H}=\{\mathbf{q}=q_1+q_2 i+q_3 j+q_4 k \mid q_1,q_2,q_3,q_4\in\mathbb{R}\}$, where the orthogonal imaginary numbers obey the following multiplicative rules[2]: $i^2=j^2=k^2=ijk=-1, ij=-ji=k, jk=-kj=i, ki=-ik=j$

Multiplications by fixed coefficients of **unit norm** quaternions expected $q=\sqrt{q_1^2+q_2^2+q_3^2+q_4^2}=1$

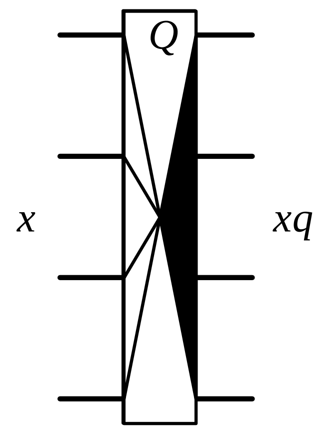
The **polar form** quaternion is $q=|q|e^{i\phi}e^{j\psi}e^{k\chi}$, where $-\pi\le\phi<\pi, -\frac{\pi}{2}\le\psi\le\frac{\pi}{2}, -\frac{\pi}{2}\le\chi\le\frac{\pi}{2}$.

Quaternion multiplication can be determined in **matrix notation**

$$qx \Leftrightarrow \underbrace{\begin{bmatrix} q_1 & -q_2 & -q_3 & -q_4 \\ q_2 & q_1 & -q_4 & q_3 \\ q_3 & q_4 & q_1 & -q_2 \\ q_4 & -q_3 & q_2 & q_1 \end{bmatrix}}_{\mathbf{M}^+(q)} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}}$$



$$xq \Leftrightarrow \underbrace{\begin{bmatrix} q_1 & -q_2 & -q_3 & -q_4 \\ q_2 & q_1 & q_4 & -q_3 \\ q_3 & -q_4 & q_1 & q_2 \\ q_4 & q_3 & -q_2 & q_1 \end{bmatrix}}_{\mathbf{M}^-(q)} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}}$$



Two different multiplication matrices: the left-operand one $\mathbf{M}^+(\cdot)$, and the right-operand $\mathbf{M}^-(\cdot)$, which related following way: $\mathbf{M}^{\mp}(q)=\mathbf{D}_C\mathbf{M}^{\pm}(\bar{q})\mathbf{D}_C$, where $\mathbf{D}_C=\mathrm{diag}(1,-\mathbf{I}_3)$ – hypercomplex **conjugate** in matrix notation.

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[2] I. L. Kantor and A. S. Solodovnikov, *Hypercomplex Numbers: an Elementary Introduction to Algebras*. New York, NY: Springer, 1989.

7. A quaternionic structure of 4-channel PMI LP PUFB

Factorization of 4-channel quaternionic linear phase filter bank PMI LP PUFBs ($\mathbf{E}(z)$ is analysis transfer matrix)[3]:

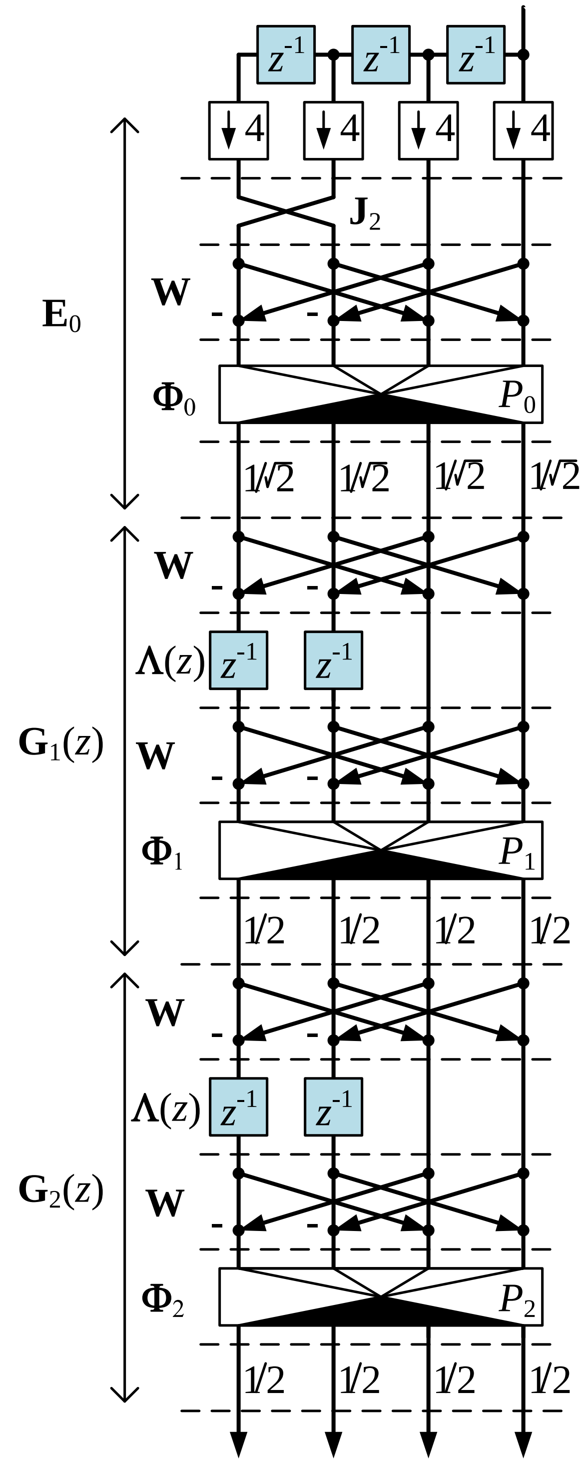
$$\mathbf{E}(z) = \left(\prod_{i=N-1}^{1} \mathbf{G}_i(z)\right) \cdot \frac{1}{\sqrt{2}} \mathbf{\Phi}_0 \mathbf{W} \cdot \mathrm{diag}(\mathbf{I}_{M/2}, \mathbf{J}_{M/2});$$

$$\mathbf{G}_i(z) = \frac{1}{2} \mathbf{\Phi}_i \mathbf{W} \cdot \mathbf{\Lambda}(z) \cdot \mathbf{W},\ \ i = 1, \ldots, N-1;$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{I}_2 & \mathbf{I}_2 \\ \mathbf{I}_2 & -\mathbf{I}_2 \end{bmatrix};\quad \mathbf{\Lambda}(z) = \mathrm{diag}(\mathbf{I}_2, z^{-1}\mathbf{I}_2);$$

$$\mathbf{\Phi}_i = \mathbf{M}^+\left(P_i\right);\ \ \mathbf{\Phi}_{N-1} = \mathbf{M}^+\left(P_{N-1}\right) \cdot \mathrm{diag}(\mathbf{J}_{M/2} \cdot \mathbf{\Gamma}_{M/2}, \mathbf{I}_{M/2}).$$

where N is order of the factorization; $\mathbf{I}_{M/2}$ and $\mathbf{J}_{M/2}$ denote the $(M/2)\times(M/2)$ identity and reversal matrices, respectively; $\mathbf{\Gamma}_{M/2}$ is diagonal matrix the elements of which are defined as $\gamma_{mm} = (-1)^{m-1},\ m = \overline{1, M-1}.$



[3] M. Parfieniuk and A. Petrovsky, "Inherently lossless structures for eight- and six-channel linear-phase paraunitary filter banks based on quaternion multipliers," *Signal Process.*, vol. 90, pp. 1755–1767, 2010.

8. 2-D non-separable PMI LP PUFB

A factorization of Q-PUBF is applied to a 2-D input signal:

$$\mathbf{y}_{n,n} = \mathbf{E}\cdot\boldsymbol{x}_{n,n}\cdot\mathbf{E}^{\mathrm{T}} = \mathbf{G}_{N-1}(\mathrm{z})\cdot\ldots\cdot\mathbf{G}_1(\mathrm{z})\cdot\mathbf{E}_0\cdot\boldsymbol{x}_{n,n}\cdot\mathbf{E}_0^{\mathrm{T}}\cdot\mathbf{G}_1^{\mathrm{T}}(\mathrm{z})\cdot\ldots\cdot\mathbf{G}_{N-1}^{\mathrm{T}}(\mathrm{z})$$

Sequence of matrix replacement for PMI LP Q-PUFB:

$$\mathbf{y}_{n,n} = \ldots\cdot\underline{\boldsymbol{\Phi}_0\cdot\underline{\mathbf{W}\cdot\underline{\mathrm{diag}\left(\mathbf{I}_{M/2},\mathbf{J}_{M/2}\right)\cdot\mathbf{x}_{n,n}\cdot\mathrm{diag}\left(\mathbf{I}_{M/2},\mathbf{J}_{M/2}\right)^{\mathrm{T}}}\cdot\mathbf{W}^{\mathrm{T}}}\cdot\boldsymbol{\Phi}_0^{\mathrm{T}}}\cdot\ldots$$

2D non-separable PMI LP Q-PUFB:

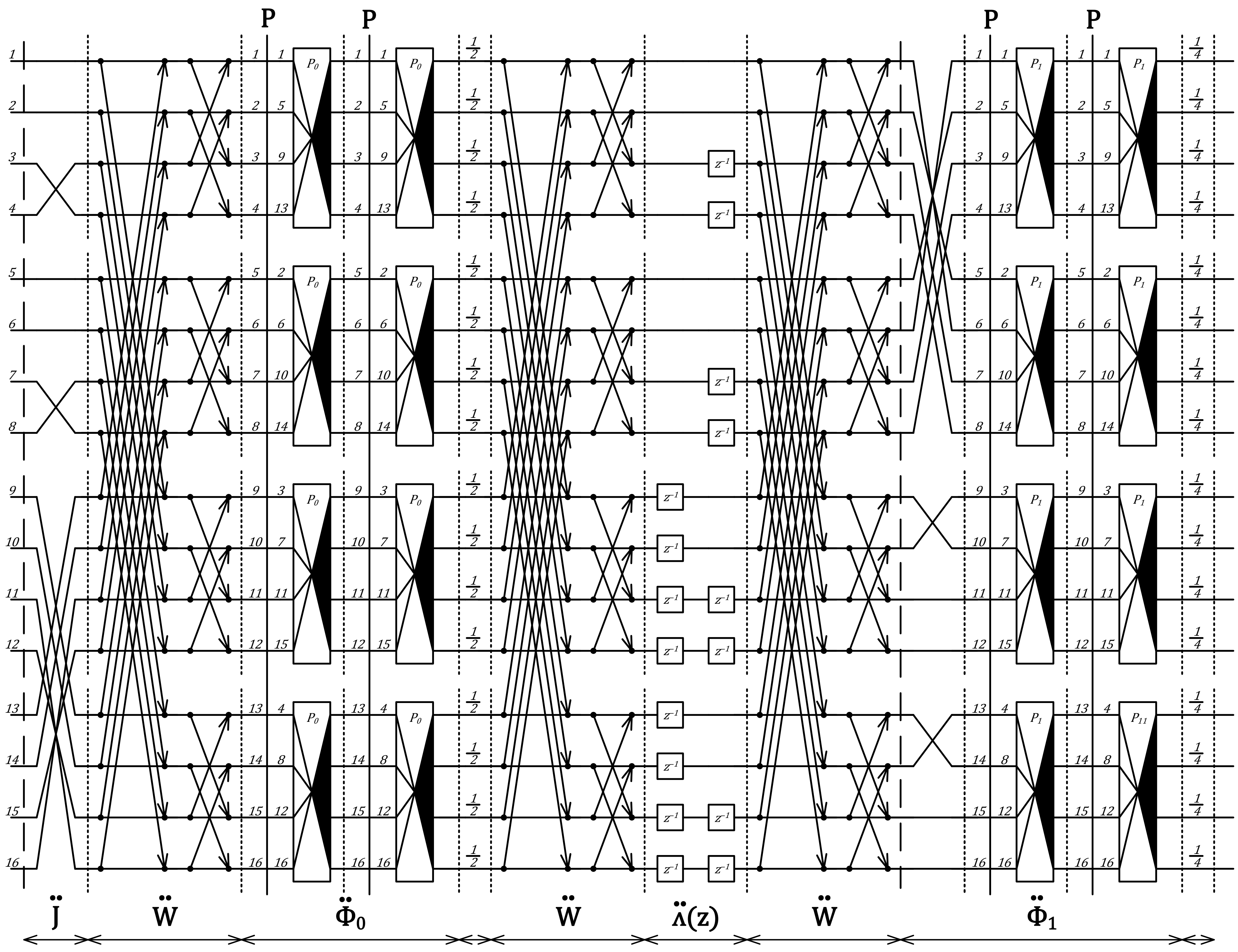
$$\mathbf{y}_{n^2,1} = \ddot{\mathbf{E}}(z)\cdot\mathbf{x}_{n\cdot n,1} = \ddot{\mathbf{G}}_{N-1}(\mathrm{z})\cdot\ddot{\mathbf{G}}_{N-2}(\mathrm{z})\cdot\ldots\cdot\ddot{\mathbf{G}}_1(\mathrm{z})\cdot\ddot{\mathbf{E}}_0\cdot\mathbf{x}_{n^2,1}$$

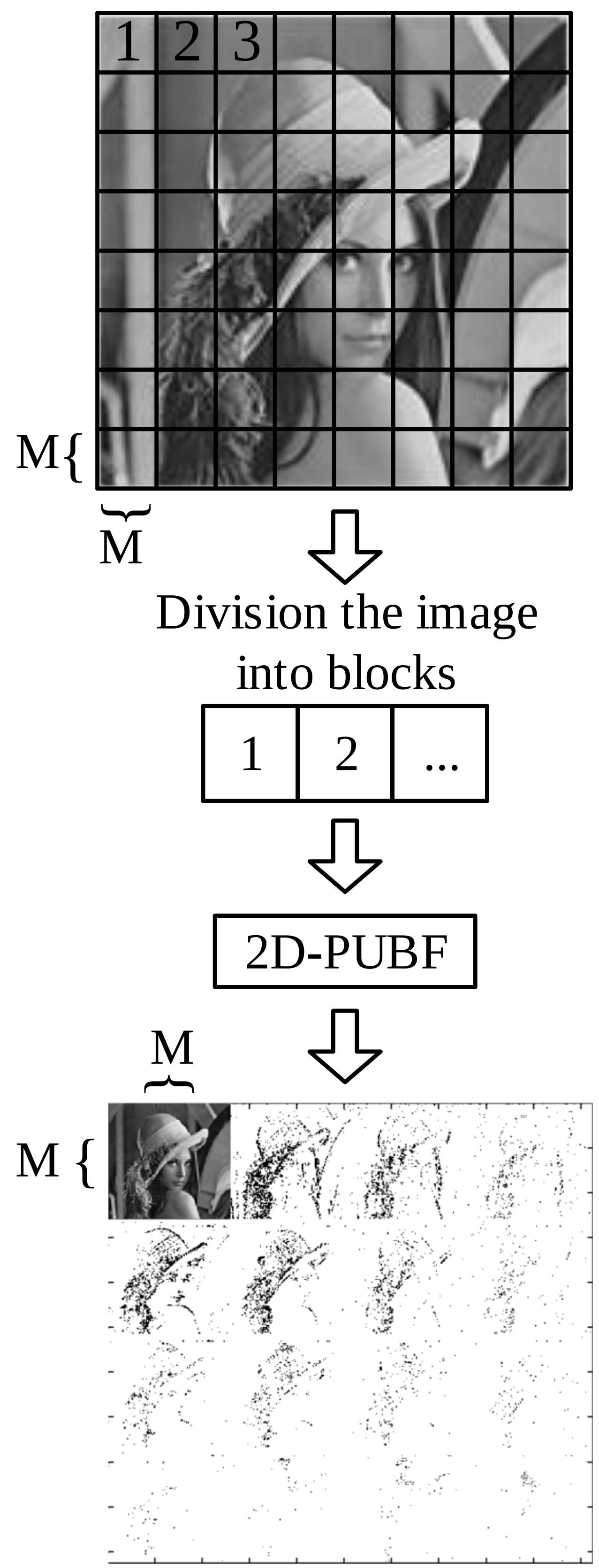
$$\ddot{\mathbf{E}}_0 = \frac{1}{2}\cdot\ddot{\boldsymbol{\Phi}}_0\cdot\ddot{\mathbf{W}}\cdot\mathfrak{D}\left(\mathrm{diag}\left(\mathbf{I}_{M/2},\mathbf{J}_{M/2}\right)\right)\cdot\mathbf{P}\cdot\mathfrak{D}\left(\mathrm{diag}\left(\mathbf{I}_{M/2},\mathbf{J}_{M/2}\right)\right)\cdot\mathbf{P}$$

$$\ddot{\mathbf{G}}_i = \frac{1}{4}\cdot\ddot{\boldsymbol{\Phi}}_i\cdot\ddot{\mathbf{W}}\cdot\ddot{\boldsymbol{\Lambda}}(z)\cdot\ddot{\mathbf{W}}.$$

where •• denotes the 2-D transformation matrix.

9. 2-D non-separable 4-channel PMI LP PUFB





10. Block Lifting factorization of $M^{+}(q)$

The **left-operand** multiplication matrix $\mathbf{M}^{+}(q)$ can be of the following structure[4]:

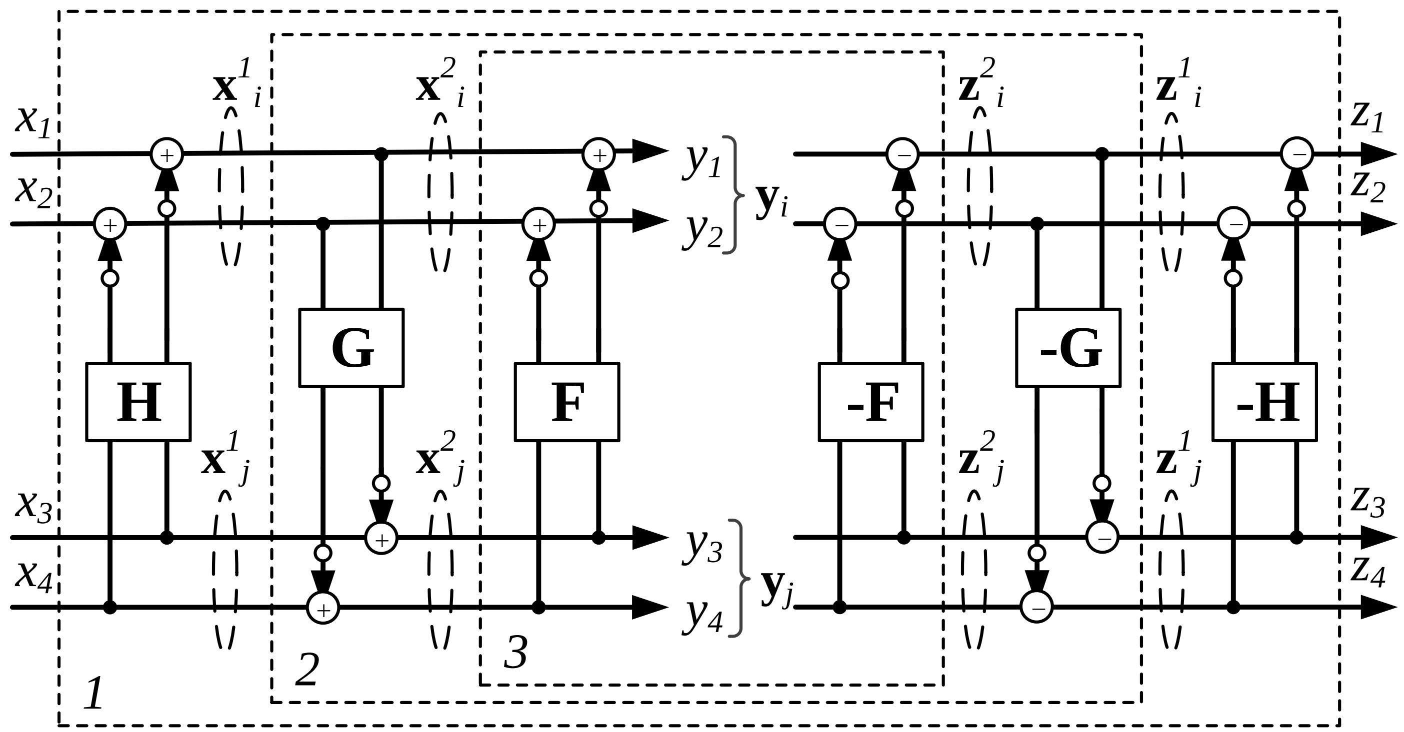
$$\mathbf{M}^{+}(Q)=\begin{bmatrix}\mathbf{C}(Q) & -\mathbf{S}(Q)\\ \mathbf{S}(Q) & \mathbf{C}(Q)\end{bmatrix};\mathbf{C}(Q)=\begin{bmatrix}q_1 & -q_2\\ q_2 & q_1\end{bmatrix};\mathbf{S}(Q)=\begin{bmatrix}q_3 & q_4\\ q_4 & -q_3\end{bmatrix};$$

$$\mathbf{F}(Q)=\left(\mathbf{C}(Q)\mp\mathbf{I}_2\right)\mathbf{S}(Q)^{-1};\mathbf{G}(Q)=\mathbf{S}(Q);\mathbf{H}(Q)=\mathbf{S}(Q)^{-1}\left(\mathbf{C}(Q)\mp\mathbf{I}_2\right).$$

The quaternion multiplication as integer-to-integer operator:

$$\mathbf{M}^{+}(Q)=\underbrace{\begin{bmatrix}\mathbf{I}_2 & \mathbf{F}(Q)\\ 0 & \mathbf{I}_2\end{bmatrix}}_{\mathbf{U}(Q)}\underbrace{\begin{bmatrix}\mathbf{I}_2 & 0\\ \mathbf{G}(Q) & \mathbf{I}_2\end{bmatrix}}_{\mathbf{L}(Q)}\underbrace{\begin{bmatrix}\mathbf{I}_2 & \mathbf{H}(Q)\\ 0 & \mathbf{I}_2\end{bmatrix}}_{\mathbf{V}(Q)};$$

$$\mathbf{M}^{+}(\overline{Q})=\underbrace{\begin{bmatrix}\mathbf{I}_2 & -\mathbf{H}(Q)\\ 0 & \mathbf{I}_2\end{bmatrix}}_{\mathbf{U}(\overline{Q})}\underbrace{\begin{bmatrix}\mathbf{I}_2 & 0\\ -\mathbf{G}(Q) & \mathbf{I}_2\end{bmatrix}}_{\mathbf{L}(\overline{Q})}\underbrace{\begin{bmatrix}\mathbf{I}_2 & -\mathbf{F}(Q)\\ 0 & \mathbf{I}_2\end{bmatrix}}_{\mathbf{V}(\overline{Q})}.$$



[4] M. Parfieniuk and A. Petrovsky, “Quaternion multiplier inspired by the lifting implementation of plane rotations,” IEEE Trans. Circuits Syst. I, vol. 57, no. 10, pp. 2708–2717, Oct. 2010.

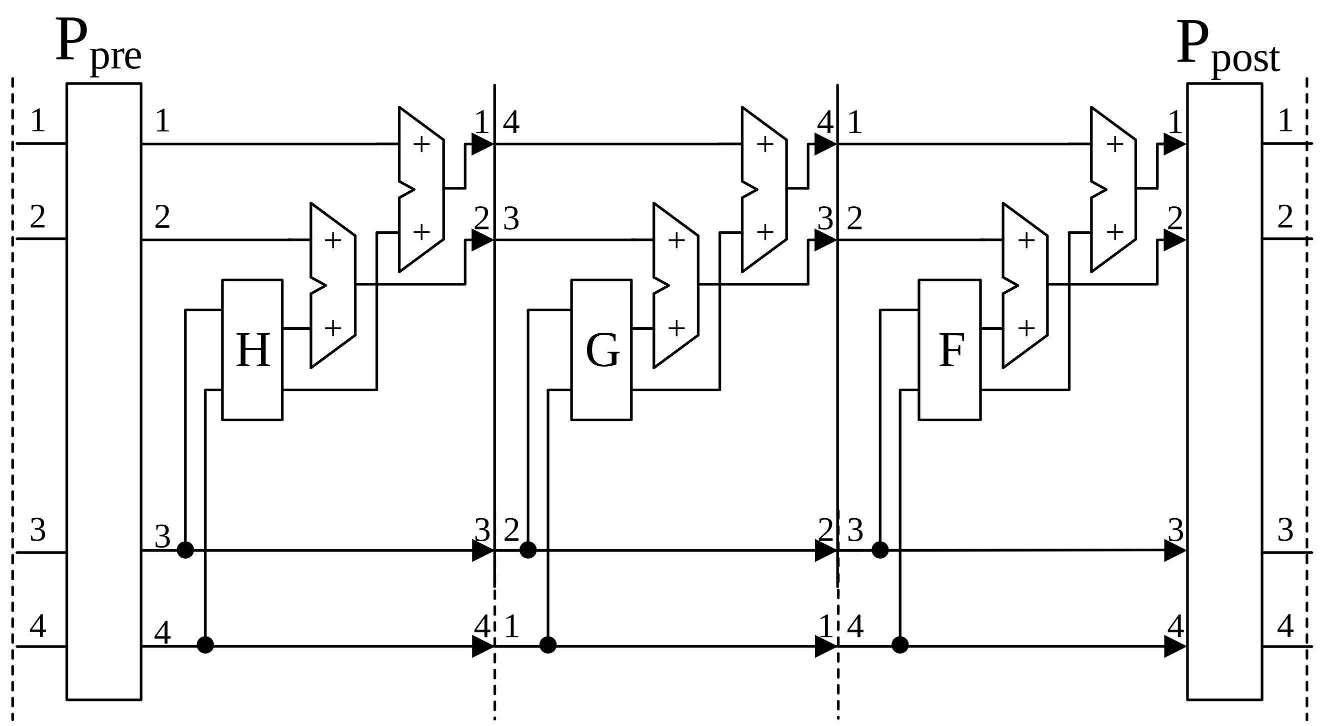
11. Controlling the dynamic range of lifting coefficients

The use of the ladder circuit parameterization increases the dynamic range of the matrix coefficients, and that is unacceptable for fixed-point arithmetic. Bringing the parameters of the multiplier to the required dynamic range can be achieved if the quaternion multiplication operator selected according to the following equation[5]:

$$\mathbf{M}^{\pm}(Q)=\begin{cases}\mathbf{P}_{post}\cdot\mathbf{M}^{\pm}(\tilde{Q})\cdot\mathbf{P}_{pre}, \text{ if } \det(\mathbf{P})=1,\\ \mathbf{P}_{post}\cdot\mathbf{M}^{\mp}(\tilde{Q})\cdot\mathbf{P}_{pre}, \text{ if } \det(\mathbf{P})=-1,\end{cases}$$

$$Qx=\mathbf{M}^{+}(Q)\mathbf{x}=\mathbf{P}_{post}\mathbf{M}^{\pm}(\tilde{Q})\mathbf{P}_{pre}\mathbf{x}=$$

$$=\mathbf{P}_{post}\underbrace{\begin{bmatrix}\mathbf{I}_2 & \mathbf{F}(\tilde{Q})\\ 0 & \mathbf{I}_2\end{bmatrix}}_{\mathbf{U}(\tilde{Q})}\underbrace{\begin{bmatrix}\mathbf{I}_2 & 0\\ \mathbf{G}(\tilde{Q}) & \mathbf{I}_2\end{bmatrix}}_{\mathbf{L}(\tilde{Q})}\underbrace{\begin{bmatrix}\mathbf{I}_2 & \mathbf{H}(\tilde{Q})\\ 0 & \mathbf{I}_2\end{bmatrix}}_{\mathbf{V}(\tilde{Q})}\mathbf{P}_{pre}\mathbf{x}$$



[5] M. Parfieniuk and A. Petrovsky, "Quaternion multiplier inspired by the lifting implementation of plane rotations," IEEE Trans. Circuits Syst. I, vol. 57, no. 10, pp. 2708–2717, Oct. 2010.

12. Universal quaternion multiplier

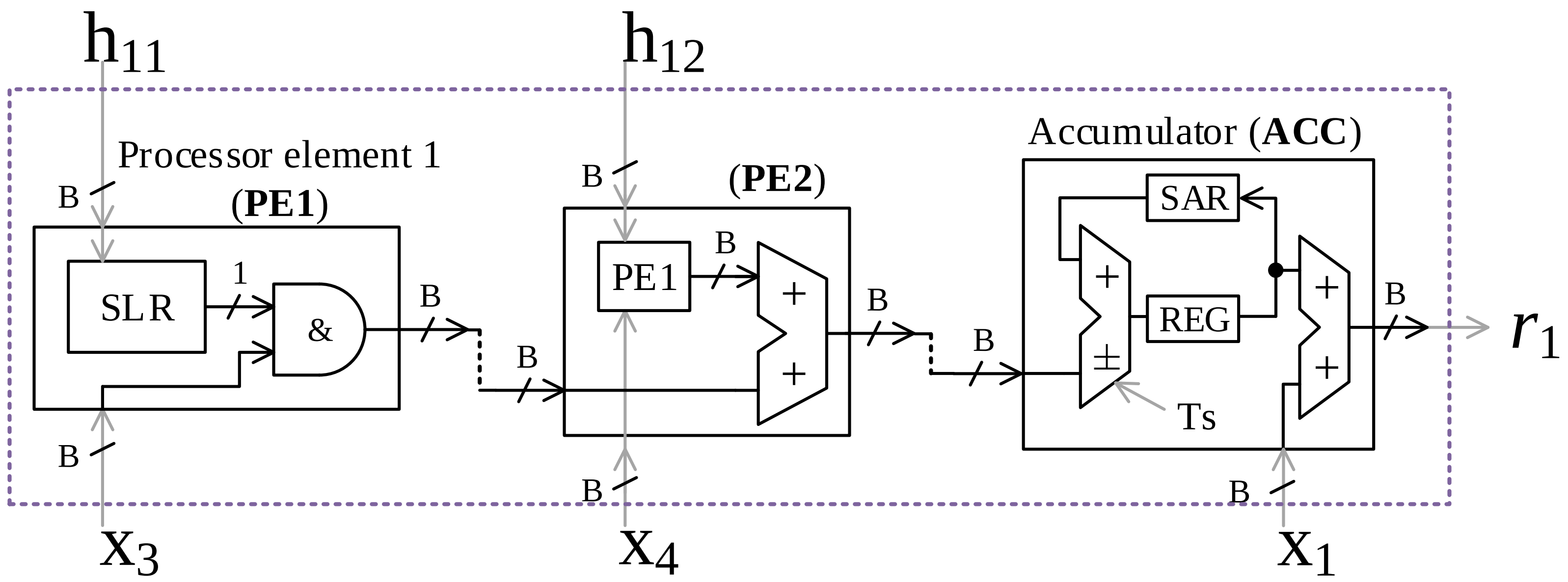
In order to unify the quaternion multiplier structure, only left multiplication $\mathbf{M}^{+}(Q)$ can be used, to adjust it to the required multiplication operator $\mathbf{M}^{+}(\overline{Q})$, $\mathbf{M}^{-}(Q)$ or $\mathbf{M}^{-}(\overline{Q})$:

Rule	Target op.	Modification rule for $\tilde{Q}$
1	$\mathbf{M}^{+}(Q)$	$\tilde{\mathbf{P}}_{pre} = \mathbf{P}_{pre}$ $\quad \tilde{\mathbf{P}}_{post} = \mathbf{P}_{post}$ $\tilde{\mathbf{F}} = \mathbf{F} \quad \tilde{\mathbf{G}} = \mathbf{G} \quad \tilde{\mathbf{H}} = \mathbf{H}$
2	$\mathbf{M}^{+}\left(\overline{Q}\right)$	$\tilde{\mathbf{P}}_{pre} = \mathbf{P}_{post}^{T}$ $\quad \tilde{\mathbf{P}}_{post} = \mathbf{P}_{pre}^{T}$ $\tilde{\mathbf{F}} = -\mathbf{H} \quad \tilde{\mathbf{G}} = -\mathbf{G} \quad \tilde{\mathbf{H}} = -\mathbf{F}$
3	$\mathbf{M}^{-}(Q)$	$\tilde{\mathbf{P}}_{pre} = \left(\mathbf{P}_{post}^{T}\right)\mathbf{D}_{c}$ $\quad \tilde{\mathbf{P}}_{post} = \mathbf{D}_{c}\left(\mathbf{P}_{pre}^{T}\right)$ $\tilde{\mathbf{F}} = -\mathbf{H} \quad \tilde{\mathbf{G}} = -\mathbf{G} \quad \tilde{\mathbf{H}} = -\mathbf{F}$
4	$\mathbf{M}^{-}\left(\overline{Q}\right)$	$\tilde{\mathbf{P}}_{pre} = \mathbf{P}_{pre} \cdot \mathbf{D}_{c}$ $\quad \tilde{\mathbf{P}}_{post} = \mathbf{D}_{c} \cdot \mathbf{P}_{post}$ $\tilde{\mathbf{F}} = \mathbf{F} \quad \tilde{\mathbf{G}} = \mathbf{G} \quad \tilde{\mathbf{H}} = \mathbf{H}$

13. Pipeline structure of the integer Q-MUL multiplier

An effective method for matrix multiplication $\tilde{\mathbf{H}}$ can be formulated in the terms of the adder-based distributed arithmetic following way:

$$r_1 = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -h_{11}^{\{0\}} \cdot x_3 & -h_{12}^{\{0\}} \cdot x_4 \\ 2^{-1} \cdot h_{11}^{\{1\}} \cdot x_3 & 2^{-1} \cdot h_{12}^{\{1\}} \cdot x_4 \\ \vdots & \vdots \\ 2^{-L} \cdot h_{11}^{\{L\}} \cdot x_3 & 2^{-L} \cdot h_{12}^{\{L\}} \cdot x_4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

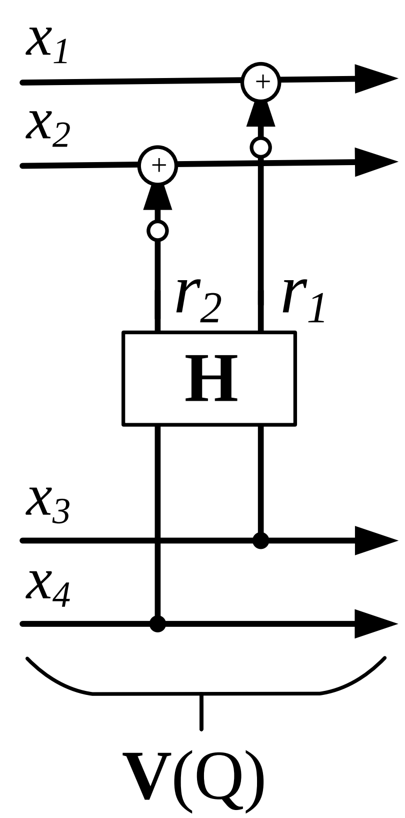


where r_i are inner products between a fixed coefficients of the block-lifting step $\mathbf{V}(Q)$ and a variable vector data $\mathbf{x}_i = [x_1 x_2]^T$ or $\mathbf{x}_j = [x_3 x_4]^T$;

$h_{ij}^{\{t\}} \in \{0,1\}$ are binary coefficients elements of the matrix $\mathbf{H}$ in 2's-complement code; ij – element idex; t – bit position;

$L = B - 1$ – less significant bit position; B – word length;

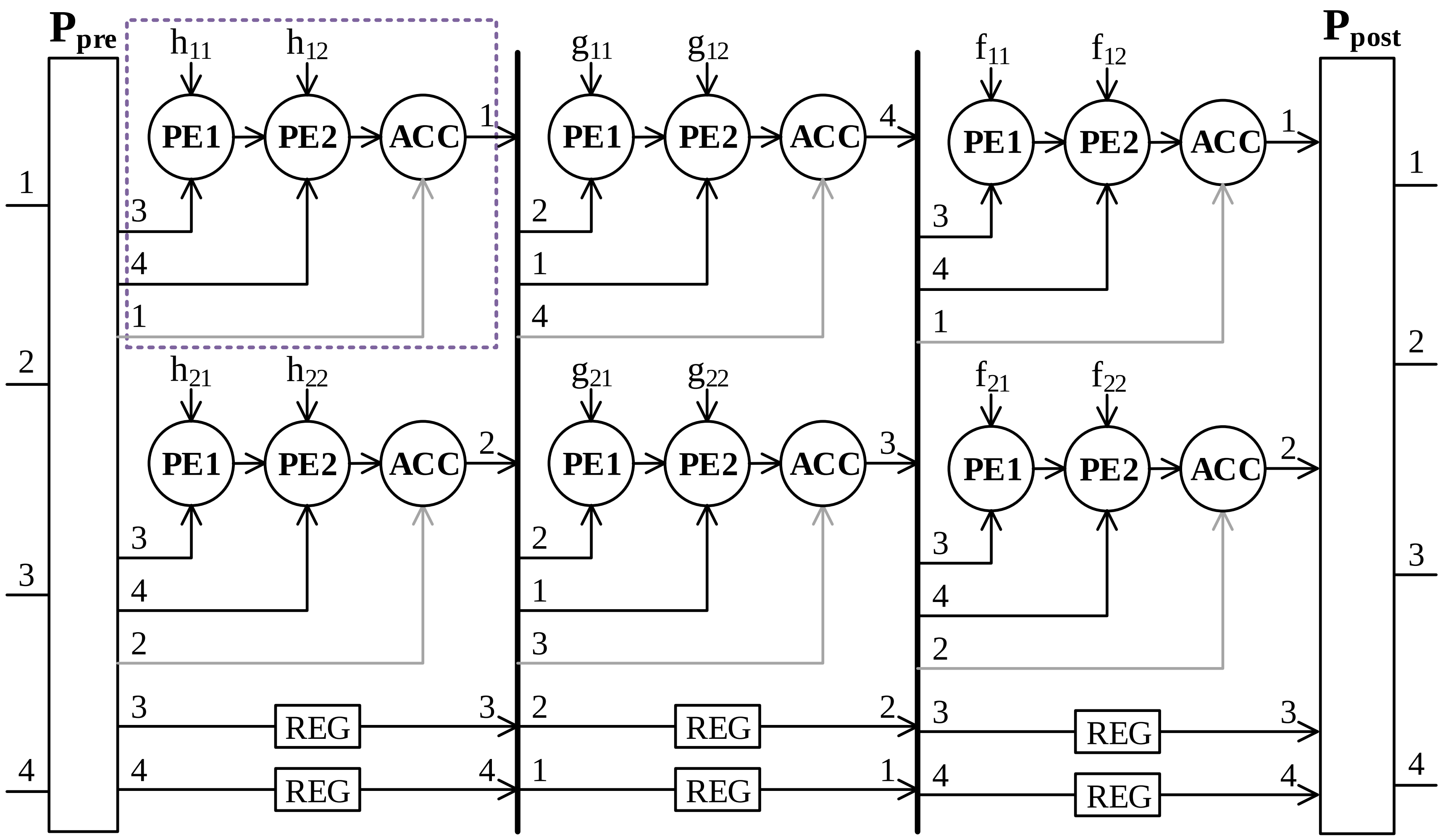
$h_{ij}^{\{0\}}$ – sign bit; Ts – signal of sign bit.



14. Pipelined embedded processor for multiplying quaternions

- The same expression can be obtained for r_2 using second row of matrix $\mathbf{H}$ and stages $\mathbf{L}(Q)$ and $\mathbf{U}(Q)$.
- The result formation take B clock cycles for all stages (where B is word length).
- The pipeline latency of Q-MUL is $3B$ clock cycles.
- The performance of pipeline is f_{CLK} / B quaternion multiplication per second.

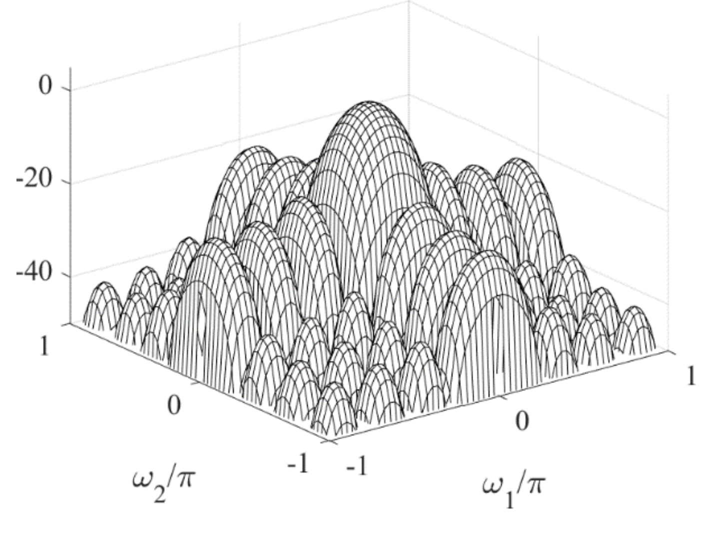
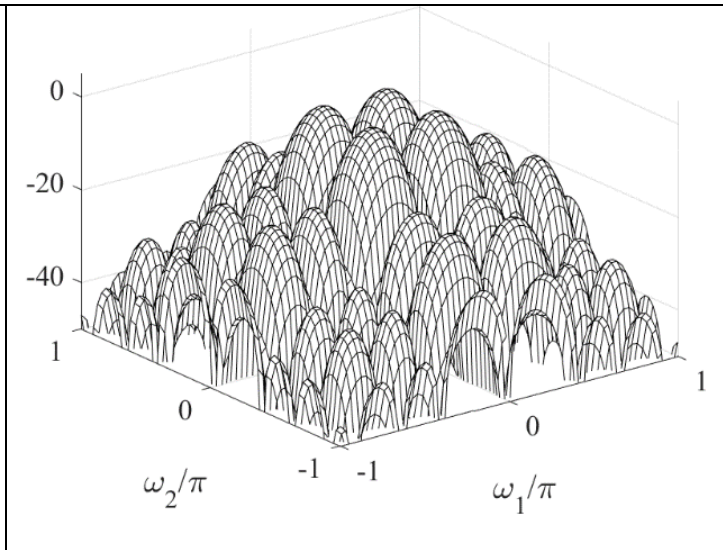
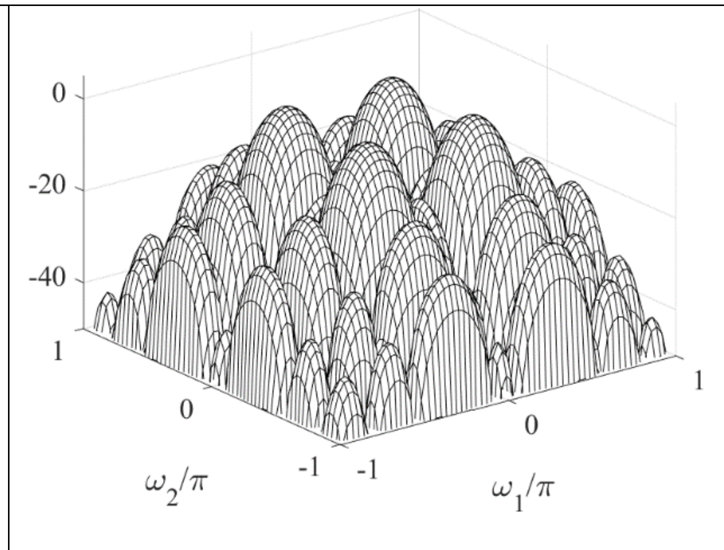
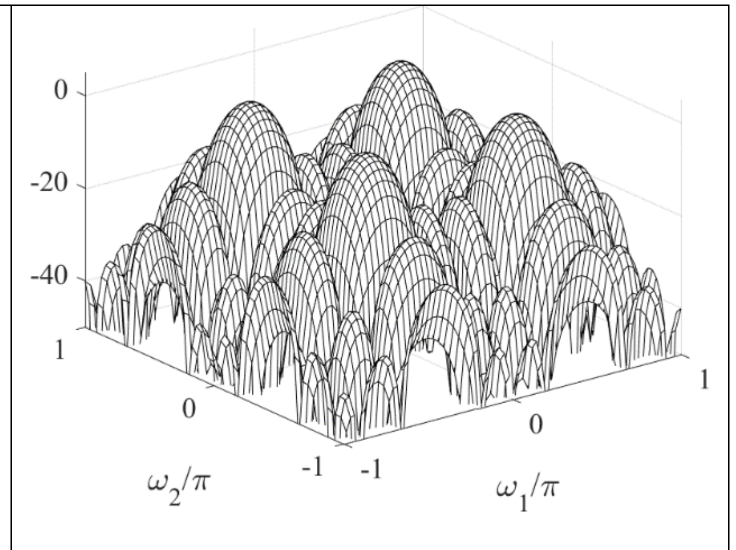
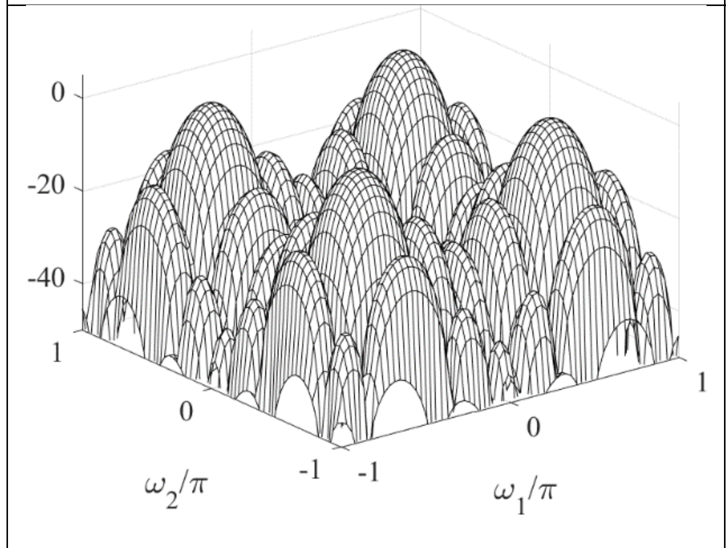
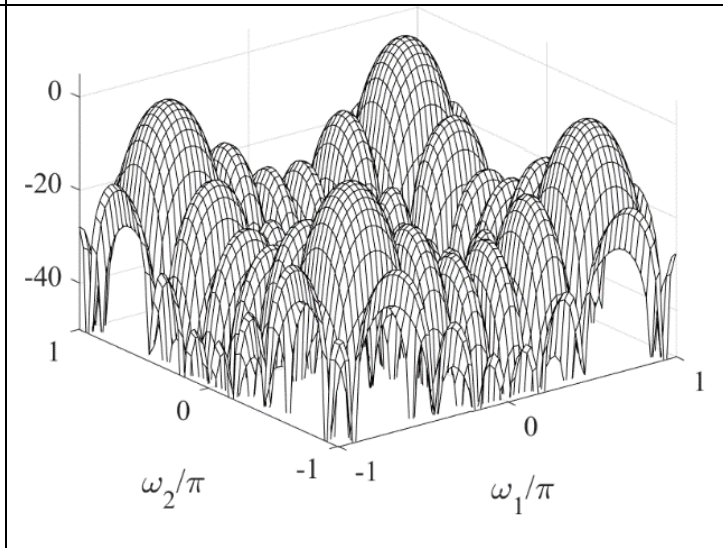
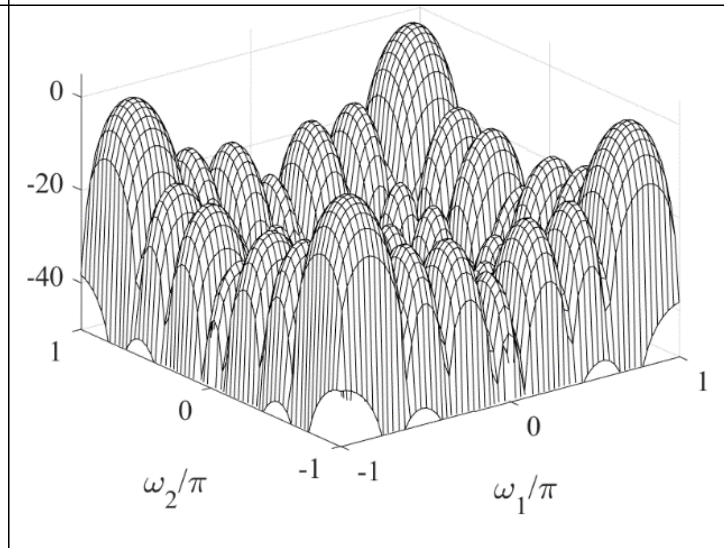
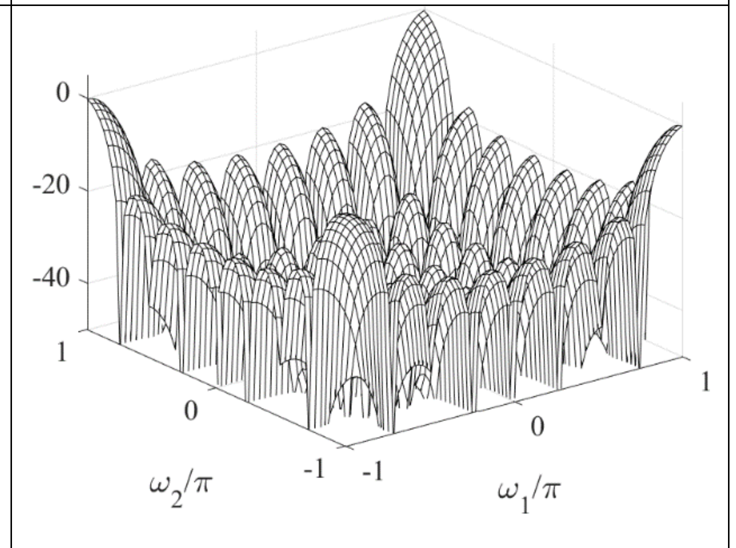
The architecture of pipelined embedded processor



$$qx = \mathbf{M}^{+}(q)\mathbf{x} = \mathbf{P}_{post}\underbrace{\begin{bmatrix} \mathbf{I}_2 & \mathbf{F}(\tilde{q}) \\ 0 & \mathbf{I}_2 \end{bmatrix}}_{\mathbf{U}(\tilde{q})}\underbrace{\begin{bmatrix} \mathbf{I}_2 & 0 \\ \mathbf{G}(\tilde{q}) & \mathbf{I}_2 \end{bmatrix}}_{\mathbf{L}(\tilde{q})}\underbrace{\begin{bmatrix} \mathbf{I}_2 & \mathbf{H}(\tilde{q}) \\ 0 & \mathbf{I}_2 \end{bmatrix}}_{\mathbf{V}(\tilde{q})}\mathbf{P}_{pre}\,\mathbf{x}$$

15. Experimental results

Magnitude responses of 2-D NS Q-PUBF.

			
(1,1)	(2,2)	(3,3)	(4,4)
			
(5,5)	(6,6)	(7,7)	(8,8)

The coding gains CG_{MD} of 2-D non-separable PMI LP Q-PUFB for the isotropic autocorrelation function with the correlation factor ρ = 0.95 are $CG_{MD} = \mathbf{13.4}\,dB(M = 4)$; $CG_{MD} = \mathbf{17.15}\,dB(M = 8)$.

16. Experimental results

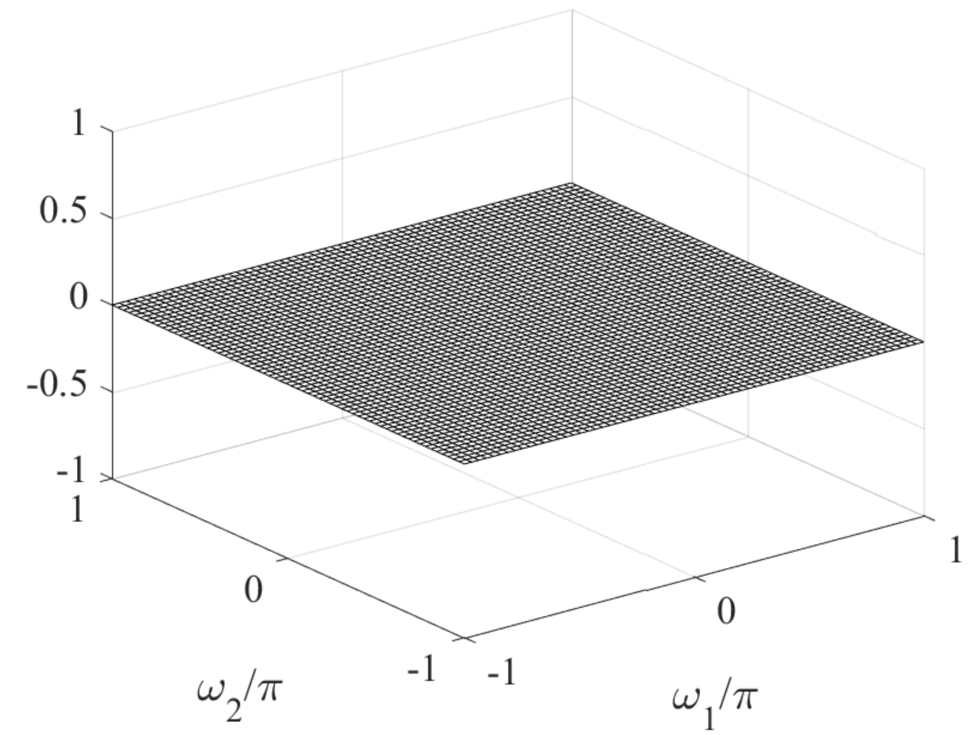
FPGA resource utilization of Q-MUL.

Xilinx ZYNQ xc7z020-1-CLG484

Slice Registers	759
Slice LUTs	560
Clock period frequency	280 MHz
Q-MUL coefficients precision	8 Bits
Input data word length	16 Bits

Total magnitude response of

analysis-synthesis system [dB].



Conclusion

- The 2-D-NSQ-PUFB based on the given QMUL is a perfect reconstruction filter bank for finite precision, compared to know solutions it have less implementation complexity, higher coding gain and stopband attenuation.
- The proposed multiplier is versatile, which allows using only $\mathbf{M}^{+}(Q)$ left multiplication matrix.
- The latency of parallel-pipeline processing does not depend on the size of the original image.